Chapter 2
Transforms

Introduction

In the Hunter Motor Controller, the basic motor controller software is designed to run a 2-phase motor, such as a 2-phase stepper motor. To run a 3-phase motor, the motor is converted in software to the equivalent 2-phase motor. The motor currents measured by the hardware are converted to the equivalent 2-phase motor currents and the output 2-phase motor voltages from the controller are converted to the equivalent 3-phase motor voltages before being applied to the motor via the Pulse Width Modulator (PWM).

Motors normally have a three-phase winding, but a two-phase winding is sometimes used, most commonly for stepping motors. The advantage of the two-phase winding, consisting of two windings separated by 90° electrical rotation, is that there is no cross-coupling between the windings. Voltages applied to one winding do not result in voltages appearing on the other winding. This is not the case for a three-phase winding. Because of the lack of cross-coupling, motor controller software is best designed to operate with a two-phase motor regardless of the actual number of motor phases. This greatly simplifies the software design.

The standard mathematical transform used to convert the measured 3-phase currents in a 3-phase motor to 2-phase currents is the Clarke transform, and the mathematical transform used to convert the 2-phase voltages to 3-phase voltages is the inverse Clarke transform. The relation between the (α and β) 2-phase values and the (u, v, and w) 3-phase values in the space vector plane is shown in Figure 2.1. Here, the α phase is aligned with the u phase, which is the most common alignment as it simplifies the conversion formulas and is the alignment used initially in the description of the transforms in this chapter. Later in this chapter, the alignment is changed by moving the α and β phases clockwise by 45 degrees. This makes the α and β phases symmetric with the u, v, and w, which helps reduce the effects of PWM distortion in the output switching pattern.

The standard Clarke transform for the currents, which includes scaling so that the output i_α and i_β waveforms have the same amplitude as the input i_u, i_v, and i_w waveforms, is:

$$
\begin{aligned}
i_\alpha &= \frac{2}{3} i_u - \frac{1}{3}(i_v + i_w) \\
i_\beta &= \frac{1}{\sqrt{3}}(i_v - i_w) \\
i_0 &= \frac{1}{3}(i_u + i_v + i_w)
\end{aligned}
\tag{2.1}
$$

Normally, the motor controller has only two current sensors, typically one in the v phase and one in the w phase. Also, the 3-phase motor is a Y connected motor with no neutral connection, giving $i_u + i_v + i_w = 0$.

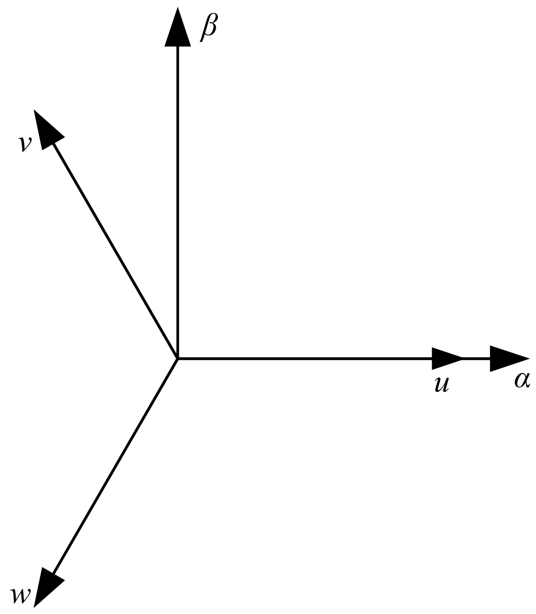

Figure 2.1. Space vector diagram of 2-phase and 3-phase co-ordinates.

Applying these limitations to equation (2.1), the equations can be simplified to:

$$i_\alpha = -i_v - i_w$$
$$i_\beta = \frac{1}{\sqrt{3}}(i_v - i_w) \qquad (2.2)$$

This is the transform normally used in 3-phase motor controllers.

For the Hunter Motor Controller, to simplify calculations in software, the Clarke transform of Equation (2.2) is not used directly in the controller but is instead scaled by $\sqrt{3}/2$. In this chapter, the modified transform is derived using the simple case of a resistive load as an example. This gives a better intuitive understanding of how the transform operates.

Two to Three Phase Transform

The transform used is designed to simplify the control of both 2-phase and 3-phase motors with the one controller.

The controller is designed to drive either a dual full-bridge inverter for 2-phase motors as shown in Figure 2.2(a) or a triple half-bridge inverter for 3-phase motors, as shown in Figure 2.2(b).

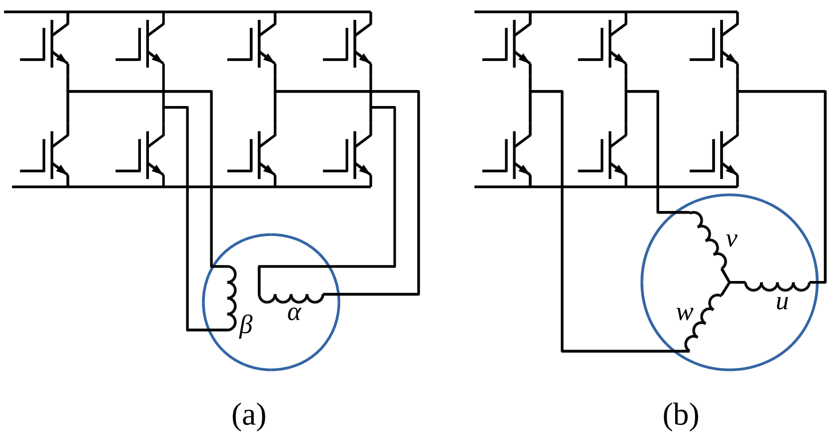

Figure 2.2. Inverter and motor set up for (a) 2-phase motors and (b) 3-phase motors.

Using a resistive load as a simple example, the two-phase load with voltage waveforms is shown in Figure 2.3.

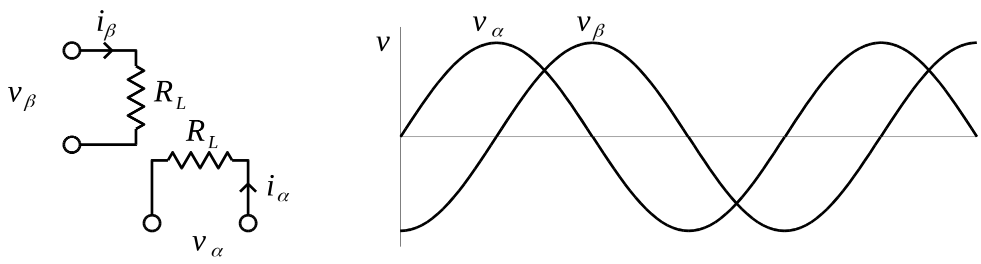

Figure 2.3. Two-phase load with voltage waveforms.

Again using a resistive load, the three-phase load with voltage waveforms is shown in Figure 2.4. The voltage waveforms are with respect to the centre point of the three-phase load.

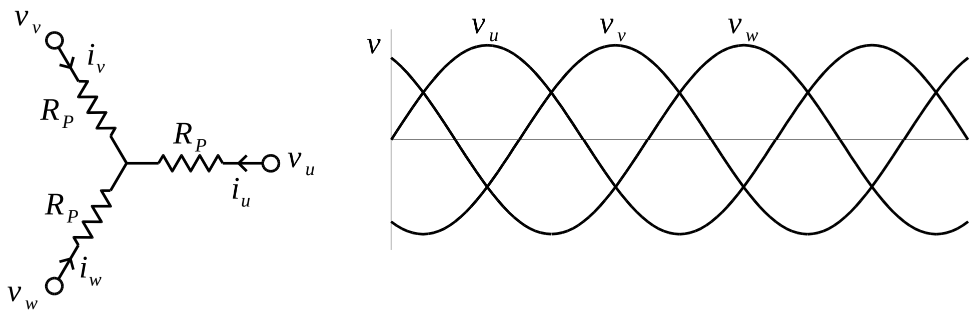

Figure 2.4. Three-phase load with voltage waveforms.

The two-phase voltage waveforms are sine waves v_α and v_β with v_β lagging v_α by 90°.

To convert from two-phase to three-phase equivalent motors, it is easiest to show how the two-phase resistive load show in Figure 2.3 can be converted to a three-phase load in a step-by-step procedure, as illustrated in Figure 2.5. The first step is to designate v_β input of the two-phase load as the the v_v-v_w input of the three phase load. Next the R_L load of the β phase is split into two, with each half set to the line to neutral three-phase resistance R_P. Next, to reduce the number of connections from 4 to 3, the lower connection of the α phase can be connected to this centre point. This will add a resistance of $R_L/4$ to the α phase, so the α phase resistance should be reduced to $3R_L/4$ to compensate. In reality, the α phase resistance must be set to $R_L/2$, not $3R_L/4$, to create the required three-phase circuit. The total α phase resistance is reduced from R_L to $3R_L/4$.

To compensate for the reduction in α phase resistance from R_L to $3R_L/4$ and keeping the same power dissipation, the measured α phase current, which is the u phase current, must be scaled down by $\sqrt{3}/2$ to get the true α phase current and the α phase voltage must be scaled down by $\sqrt{3}/2$ to get the actual α phase voltage which is the difference between the u phase voltage and the average of the v and w phase voltages. This is all illustrated in Figure 2.5 together with typical two-phase and three-phase voltage waveforms.

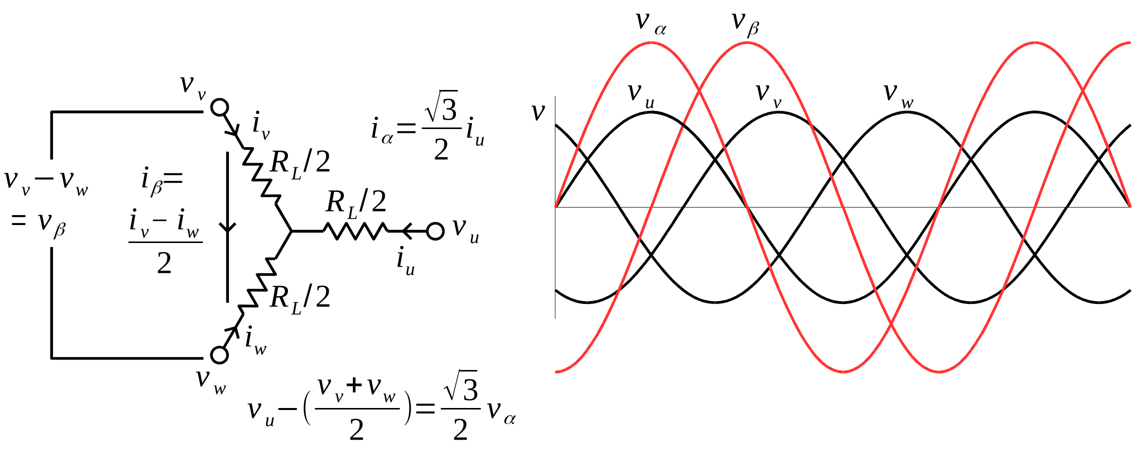

Figure 2.5. Two phase to three phase conversion.

The three-phase to two-phase current transform is:

$$i_\alpha=\frac{\sqrt{3}}{2}i_u$$
$$i_\beta=\frac{1}{2}(i_v-i_w) \tag{2.3}$$

For a motor drive with two current sensors in the v and w phases, i_u would be replaced with $-i_v$-i_w.

From Figure 2.5, the following relations for the phase voltages hold:

$$v_u-(\frac{v_v+v_w}{2})=\frac{\sqrt{3}}{2}v_\alpha \tag{2.4}$$

$$v_v-v_w=v_\beta \tag{2.5}$$

To set the neutral voltage to zero, the following relation is required:

$$v_u+v_v+v_w=0 \tag{2.6}$$

For the pulse width modulated output, the neutral voltage is adjusted to minimise the peak phase leg voltages to generate the maxim output voltage for a given inverter DC link voltage. This is described in Chapter 3 on the Pulse Width Modulator.

Using the zero neutral voltage of equation (2.6) relation in equation (2.4):

$$v_u = \frac{1}{\sqrt{3}} v_\alpha \tag{2.7}$$

Using equations (2.5-7):

$$v_v = -\frac{1}{2\sqrt{3}} v_\alpha + \frac{1}{2} v_\beta \tag{2.8}$$

$$v_W = -\frac{1}{2\sqrt{3}} v_\alpha - \frac{1}{2} v_\beta \tag{2.9}$$

The waveforms shown in Figure 2.5 are sinusoidal for illustration, but the conversion is for the passive circuit and holds for any applied waveforms. Also, to illustrate the conversion process a resistive load is used, but the conversion holds for loads containing reactive impedances and back emf's including unbalanced loads. Unbalanced three-phase load impedances will convert to unequal two-phase impedances.

The actual conversion can be applied to an unbalanced load, but for the design of the controller which uses the two-phase equivalent circuit, a balanced load is always assumed.

Axes Re-alignment to Balance PWM Distortion

Distortion occurs on each of the u, v, and w outputs due to pulse width modulation errors. This is mainly due to the required insertion of a delay called the dead band between turning off one switch in an inverter leg and turning on the opposing switch.

The controller is very tolerant to output distortion provided it is balanced between the α and β phases, but it is not very tolerant to unbalanced distortion, particularly distortion causing differences in amplitude between the two phases. This is interpreted as torque ripple by the controller, resulting in more current distortion and torque ripple from over-compensation.

With the alignment of the two axes systems shown in Figure 2.1, the α phase is most sensitive to distortion in the *u* output, while the β phase is mainly sensitive to differences between the *v* and *w* outputs. By rotating the $\alpha\,\beta$ axes by 45°, as shown in Figure 2.6, the sensitivity of these axes to distortion in the *u*, *v*, and *w* outputs becomes balanced, resulting in similar distortion levels in the two axes.

If the original axes are designated $\alpha'\,\beta'$, the matrix calculation to rotate the currents by -90° is:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} i_{\alpha'} \\ i_{\beta'} \end{bmatrix} \tag{2.10}$$

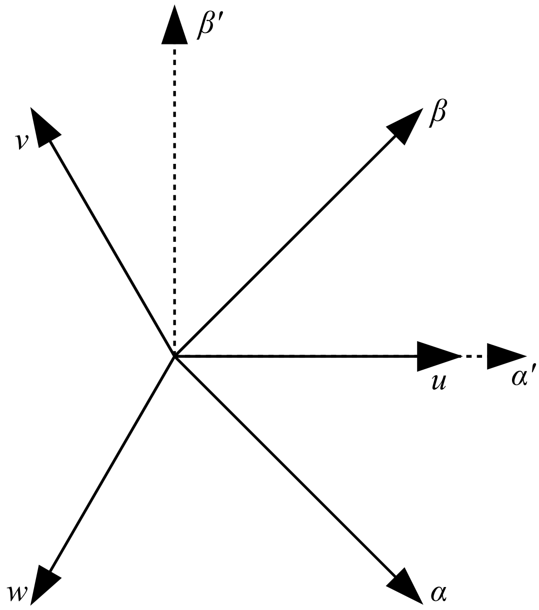

Figure 2.6. Rotation of α and β phases to balance PWM distortion.

Applying this rotation to equation (2.3):

$$i_\alpha=\frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}i_u\right)-\frac{1}{\sqrt{2}}\left(\frac{1}{2}(i_v-i_w)\right)$$
$$i_\beta=\frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}i_u\right)+\frac{1}{\sqrt{2}}\left(\frac{1}{2}(i_v-i_w)\right) \quad (2.11)$$

which re-arranges to:

$$i_\alpha=\frac{\sqrt{3}}{\sqrt{8}}i_u-\frac{1}{\sqrt{8}}(i_v-i_w)$$
$$i_\beta=\frac{\sqrt{3}}{\sqrt{8}}i_u+\frac{1}{\sqrt{8}}(i_v-i_w) \quad (2.12)$$

Using the relation:

$$i_u=-i_v-i_w \quad (2.13)$$

Equation 2.12 for *v* and *w* phase current sensors becomes:

$$i_\alpha=-\frac{\sqrt{3}+1}{\sqrt{8}}i_v-\frac{\sqrt{3}-1}{\sqrt{8}}i_w$$
$$i_\beta=-\frac{\sqrt{3}-1}{\sqrt{8}}i_v-\frac{\sqrt{3}+1}{\sqrt{8}}i_w \quad (2.14)$$

Alternatively, directly using the vector diagram of Figure 2.6:

$$i_\alpha=-\cos\left(\frac{\pi}{12}\right)i_v-\cos\left(\frac{5\pi}{12}\right)i_w$$
$$i_\beta=-\cos\left(\frac{5\pi}{12}\right)i_v-\cos\left(\frac{\pi}{12}\right)i_w \quad (2.15)$$

For the output voltages, the $\alpha\,\beta$ axes must also be rotated. With the old $\alpha\,\beta$ axes as used in equations (2.7-9) replaced with the $\alpha'\,\beta'$ axes, the old voltage values must be replaced with the following new values:

$$\begin{bmatrix} v_\alpha' \\ v_\beta' \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} \tag{2.16}$$

With these replacements, equations (2.7-9) become:

$$\begin{aligned} v_u &= \frac{1}{\sqrt{6}} v_\alpha + \frac{1}{\sqrt{6}} v_\beta \\ v_v &= -\frac{1}{2\sqrt{3}} \left(\frac{1}{\sqrt{2}} v_\alpha + \frac{1}{\sqrt{2}} v_\beta\right) + \frac{1}{2}\left(-\frac{1}{\sqrt{2}} v_\alpha + \frac{1}{\sqrt{2}} v_\beta\right) \\ v_W &= -\frac{1}{2\sqrt{3}} \left(\frac{1}{\sqrt{2}} v_\alpha + \frac{1}{\sqrt{2}} v_\beta\right) - \frac{1}{2}\left(-\frac{1}{\sqrt{2}} v_\alpha + \frac{1}{\sqrt{2}} v_\beta\right) \end{aligned} \tag{2.17}$$

which re-arranges to:

$$\begin{aligned} v_u &= \frac{1}{\sqrt{6}} v_\alpha + \frac{1}{\sqrt{6}} v_\beta \\ v_v &= -\frac{\sqrt{3}+1}{\sqrt{3}\sqrt{8}} v_\alpha + \frac{\sqrt{3}-1}{\sqrt{3}\sqrt{8}} v_\beta \\ v_W &= \frac{\sqrt{3}-1}{\sqrt{3}\sqrt{8}} v_\alpha - \frac{\sqrt{3}+1}{\sqrt{3}\sqrt{8}} v_\beta \end{aligned} \tag{2.18}$$

Alternatively, using direct translation from the vector diagram of Figure 2.6:

$$\begin{aligned} v_u &= \frac{1}{\sqrt{3}} (\cos(\pi/4) v_\alpha + \cos(\pi/4) v_\beta) \\ v_v &= \frac{1}{\sqrt{3}} (-\cos(\pi/12) v_\alpha + \cos(5\pi/12) v_\beta) \\ v_W &= \frac{1}{\sqrt{3}} (\cos(5\pi/12) v_\alpha - \cos(\pi/12) v_\beta) \end{aligned} \tag{2.19}$$

To reduce the peak output voltages to a minimum, the output voltages are modified further before being sent to the pulse width modulator by adding a non-zero neutral voltage. This is described in detail in Chapter 3.

Fixed to Rotating Co-ordinate Transform

For field oriented controllers as is this one, the currents i_α and i_β in fixed coordinates must be converted to their field oriented rotating co-ordinate values i_d and i_q. The relation between the two coordinate systems is shown in Figure 2.7.

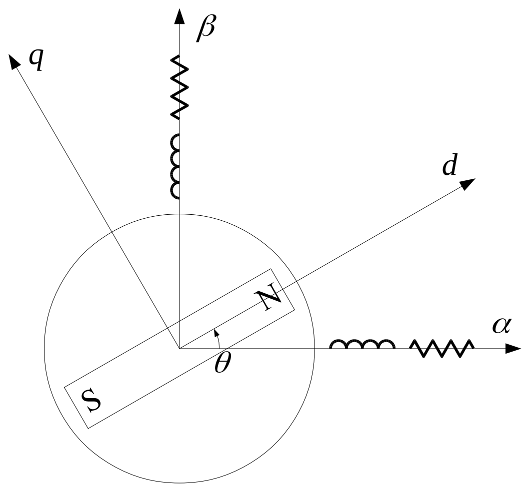

Figure 2.7. Diagram showing the fixed and rotating co-ordinates of the two-phase, two-pole equivalent PMSM.

The currents are converted from the fixed to rotating co-ordinates with the formula:

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \tag{2.20}$$

As will be described in Chapter 3 on the Feed Forward Converter, there is no corresponding d q to α β conversion required for voltages.

Controller Software Design Considerations

The two to three phase transforms were chosen to simplify the design of the controller software as much as possible especially considering the need to design the controller to accommodate both 2-phase and 3-phase motors.

With the chosen transforms, the 3-phase line to line parameters become the equivalent 2-phase parameters. The 3-phase line to line inductance, resistance, and back emf become the per phase inductance, resistance, and back emf of the equivalent 2-phase machine. Also, the peak output voltage before clipping of the 3-phase motor are the same as the peak output before clipping of the equivalent 2-phase motor, resulting in no changes needed to the basic 2-phase controller software design when swapping between 2-phase and 3-phase motors.

Current Sensor Calibration

The controller is not sensitive to overall incorrect current measurements up to at least 10%, but it can be very sensitive to differential current measurement errors for some motors. If the current measurement scaling of the α and β phases of the equivalent 2-phase machine do not closely match, the resulting modulation of the d and q axis currents is interpreted by the controller as a torque modulation, which can cause erratic results when the controller tries to compensate for this perceived modulation. This occurs at very low speeds where resistance effects predominate, particularly with high inductance motors such as stepping motors.

The solution is to calibrate the α and β phase currents so that the α and β phase motor resistances measure the same value. This can be done by applying the same fixed DC voltage to each of the α and β phases in sequence and adjust the scaling of one of the α or β phase current measurements so that the resulting α and β phase current measurements are equal. This method will also automatically compensate for slight differences between motor windings.